Hertentamen 2 Kansrekening – Open book

Tuesday 23 January 2007

1. An insurance company has observed that on the average 0.1 percent of the customers are involved in an accident of a specific type. How big is the probability that in a given year not more than 3 of the 5000 customers are involved in an accident of this type?

2. Prove that
$$\sum_{k=0}^{n} k \binom{n}{k} = n2^{n-1}$$

   Can you give a proof without computation?

3. A patient has one of the diseases A_1, A_2, A_3. Suppose that the population percentages suffering from these diseases are in the ratio $2 : 1 : 1$. The patient is given a test which turns out to be positive in 25 percent of the cases of A_1, 50 percent of the cases of A_2, and 90 percent of the cases of A_3. Given that out of three (independent) tests taken by the patient precisely 2 were positive, find the probability that the patient has the disease A_i for $i = 1, 2, 3$.

4. Let G_i be independent random variables with Normal (Gaussian) distribution with mean zero and variance 1.

   Define the random variables $S_n = \sum_{i=1}^{n} \frac{G_i}{3^i}$, for each natural number n. Find

   a) the exponential moment generating function $t \mapsto \mathbf{E}(e^{S_n t})$
   b) the variance of S_n
   c) $\mathbf{E}(S_n^4)$.
   d) Which results do you get for b) and c) in the limit $n \uparrow \infty$?

5. Let X_1 and X_2 be two random variables that are uniformly distributed on the interval $[0, 1]$.

   Find the densities of the random variables given by

   a) $X_1 + X_2$
   b) $\log X_1$
   c) $\log X_1 + \log X_2$
   d) $X_1 X_2$
   e) $\sqrt{X_1 X_2}$